EECS-70: Discrete Mathematics and Probability Theory **Spring 2020**

Topic Specific Review: Bayesian Inference

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1.1 Medical Testing

1.1.1 a

Suppose, 1 in 1000 citizens have a novel sickness, and there is a test you can use for the illness (for a price of course!) The makers advertise that the test is 99 percent reliable. They explain that healthy patients come back with a negative test exactly 99 percent of the time. Truly sick patients results come back with a positive test 99 percent of the time. A randomly selected citizen came back positive. What is the probability this randomly selected citizen is actually infected.

1.1.2 b

The citizen was woefully unsatisfied with how unconvincing his "positive" test results were in part a, given the cost of treatment. So s/he takes the test a second time, and it comes back positive. What is the probability that the citizen now actually has the disease?

1.1.3 c

(Open Ended, don't answer numerically): Suppose you felt very ill and demonstrated symptoms of this novel disease and decided to get tested for the disease, and it came back positive. How sure can you be that you have the disease? Is it the same probability as your answer to a?

1.1.4 d

(Challenge) In healthy citizens the amount of a certain protein is normally distributed $N(\mu_0 = 100, \sigma^2 = 100)$ but in infected individuals the distribution is better modeled by $N(\mu_1 = 160, \sigma^2 = 100)$. Now a randomly selected citizen is administered a new, more sophisticated (ideal) test which measures an exact protein level of 170, what is the probability the citizen has the illness, if 1/1000 citizens have the illness?
Hint: Use, without proof, the fact the probability density distribution of a guassian is given by.

$$f_x(x) = 1/sqrt(2 * \pi * \sigma^2) * e^{-\frac{(x-\mu)^2}{2*\sigma^2}}$$

Hint: Let X denote the patients protein level, and consider:

$$P(x < X < x + \epsilon | sick)$$

And then consider the limiting behavior as ϵ becomes small.